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Theoretical foundation of a decision network for urban development*

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Abstract: Planning problems are challenging and complex in that they usually involve multiple stakeholders with multi-attribute preferences. Thus few, if any, planning tools are useful in helping planners to address such problems. Decision analysis is less useful than expected in dealing with planning problems because it overwhelmingly focuses on making single decisions for a particular decision maker. This paper describes the theoretical foundation of a planning tool called Decision Network, which aims to help planners to make multiple, linked decisions when faced with multiple stakeholders with multi-attribute preferences. The research provides a starting point for a fully-fledged technology that will be useful for dealing with complex planning problems. We first provide a general formulation of the planning problem that Decision Network intends to address. We then introduce an efficient solution algorithm for this problem, with a numerical example to demonstrate how the algorithm works. The proposed solution algorithm is efficient, allowing computerization of the planning tool. We also demonstrate that the diagrammatic representation of Decision Network is more efficient than that of a decision tree. Therefore, when dealing with challenging, complex planning problems, using Decision Network to make multiple, linked decisions may yield more benefits than making such decisions independently.

Key words: Decision making; Linked decisions; Decision Network; Planning
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1 Introduction

Planning is an important research topic in the field of artificial intelligence (Ghallab *et al*., 2004; Pollock, 2006). However, in real world situations, most planning problems are complex and challenging, not only because the problems themselves are ambiguous and difficult to define, but also because they involve multiple stakeholders with multi-attribute preferences. These preferences are difficult to elicit, and planners have to make more than one decision at a time, contradicting the view that making linked decisions is rare (Keeney, 2004). In short, planning problems are ill-defined (Hopkins, 1984); therefore, their solution algorithm must be different from those of well-defined problems. Traditional techniques developed in decision analysis, such as the decision tree, focus on single decision makers with unidimensional attributes, such as utility, evaluating a given set of alternatives to select the best. In real situations, the complex nature of planning problems renders such techniques less useful than expected in helping decision makers figure out what to do. As an alternative to traditional techniques, we propose here a theoretical foundation for a technique called Decision Network for making multiple, linked decisions that involve multiple stakeholders with multi-attribute preferences. Decision Network is derived mainly from the ideas of

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the decision tree (Kirkwood, 1996), the strategic choice approach (Friend and Hickling, 2005), and the garbage can model (Cohen *et al*, 1972). A detailed description of the conceptual framework of Decision Network was provided by Han and Lai (2011), together with a description of its application to the management of urban growth boundaries (Han and Lai, 2012). Here, we provide a formal, general formulation of that framework and describe a solution algorithm for that formulation.

This paper is structured as follows: Section 2 gives the formulation of Decision Network. In Section 3, we show an efficient solution algorithm for that formulation, followed in Section 4 by a numerical example to demonstrate how the algorithm works. In Section 5, we compare the decision tree model with Decision Network in representing linked decisions, and discuss possible extensions of the current formulation. In Section 7, we provide conclusions.

2 Formulation

Consider m decision makers eligible to attend any of n decision situations. *Decision makers* are individuals or coherent groups with authority and capability to make decisions. *Decision situations* are the choice opportunities in which problems and solutions are discussed and evaluated by the decision maker(s). Assume that there are p problems and q solutions under consideration. A *utility* is the affected individual's (or stakeholder's) level of satisfaction imposed by decision makers, problems, and solutions in a decision situation. Whether a decision is made in a decision situation depends on whether the amount of utility supplied exceeds the utility demanded by that decision situation. Thus, utility can be either positive or negative (disutility). Note that decision situations can be either *deterministic* (denoted as decision nodes) or *stochastic* (denoted as chance nodes), meaning that some decision situations are pre-determined, whereas others are probabilistic in occurrence. Given these definitions, we can define the variables and parameters of the Decision Network problem as shown in Table 1.

With these variables and parameters, there are three structures in a Decision Network: namely, the decision structure, access structure, and solution structure. All the structures are represented by 0-1 matrices. The only difference is that the rows in these matrices are decision makers, problems, and solutions, respectively, with the column being decision situations. A "1" in these matrices implies that the element in the row is related to the decision situation in the corresponding column, whereas a "0" means that no such relationship exists.

Table 1 Definition of variables and parameters of the formulation

Terminology	Notation	Probability	Utility
Decision situations			
Decision node 1	d_1	1.0	not applicable
Decision node 2	d_2	1.0	not applicable
Decision node 3	d_3	1.0	not applicable
⋮	⋮	⋮	⋮
Decision node *i*	d_i	1.0	not applicable
Chance node *i*+1	d_{i+1}	p_{i+1}	not applicable
Chance node *i*+2	d_{i+2}	p_{i+2}	not applicable
Chance node *i*+3	d_{i+3}	p_{i+3}	not applicable
⋮	⋮	⋮	⋮
Chance node *n*	d_n	p_n	not applicable
Decision makers			
Decision maker 1	m_1	not applicable	u_1
Decision maker 2	m_2	not applicable	u_2
Decision maker 3	m_3	not applicable	u_3
⋮	⋮	⋮	⋮
Decision maker m	m_m	not applicable	u_m
Problems			
Problem 1	r_1	not applicable	v_1
Problem 2	r_2	not applicable	v_2
Problem 3	r_3	not applicable	v_3
⋮	⋮	⋮	⋮
Problem *p*	r_p	not applicable	v_p
Solutions			
Solution 1	s_1	not applicable	w_1
Solution 2	s_2	not applicable	w_2
Solution 3	s_3	not applicable	w_3
⋮	⋮	⋮	⋮
Solution *q*	s_q	not applicable	w_q

For example, in the decision structure, if the value of the cell in row three (e.g., a planner) and column five (e.g., a public hearing) is 1, it means that the planner is eligible to participate in the public hearing for decision making. The generic forms of the three matrices are shown in Tables 2 to 4.

Table 2 The 0-1 matrix for decision structure

Notation	d_1	d_2	d_3	$\cdots$	d_n
m_1	a_{11}	a_{12}	a_{13}	$\cdots$	a_{1n}
m_2	a_{21}	a_{22}	a_{23}	$\cdots$	a_{2n}
m_3	a_{31}	a_{32}	a_{33}	$\cdots$	a_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
m_m	a_{m1}	a_{m2}	a_{m3}	$\cdots$	a_{mn}

Table 3 The 0-1 matrix for access structure

Notation	d_1	d_2	d_3	$\cdots$	d_n
r_1	b_{11}	b_{12}	b_{13}	$\cdots$	b_{1n}
r_2	b_{21}	b_{22}	b_{23}	$\cdots$	b_{2n}
r_3	b_{31}	b_{32}	b_{33}	$\cdots$	b_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
r_p	b_{p1}	b_{p2}	b_{p3}	$\cdots$	b_{pn}

Table 4 The 0-1 matrix for solution structure

Notation	d_1	d_2	d_3	$\cdots$	d_n
s_1	c_{11}	c_{12}	c_{13}	$\cdots$	c_{1n}
s_2	c_{21}	c_{22}	c_{23}	$\cdots$	c_{2n}
s_3	c_{31}	c_{32}	c_{33}	$\cdots$	c_{3n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
s_q	c_{q1}	c_{q2}	c_{q3}	$\cdots$	c_{qn}

Note that $a_{ij} \in \{0,1\}$, for $i = 1, 2, 3, \ldots$, m and $j = 1, 2, 3, \ldots, n$; that $b_{ij} \in \{0,1\}$, for $i = 1, 2, 3, \ldots, p$ and $j = 1, 2, 3, \ldots, n$; and that $c_{ij} \in \{0,1\}$, for $i = 1, 2, 3, \ldots, q$ and $j = 1, 2, 3, \ldots, n$.

The variables and parameters in Table 1, together with the structural constraints specified in Tables 2 to 4, form the basic information for the Decision Network problem, which can be represented in a directed graph, as demonstrated by Han and Lai (2011) in a numerical example.

The task is then to make a "plan" by assigning the given m decision makers, p problems, and q solutions to n decision situations to yield the highest overall expected utility under the structural constraints. Mathematically, this assignment task can be easily formulated as a 0-1 integer program, as shown below:

$$\text{Max} \sum\nolimits_{j=1}^{n} p_j \left(\sum\nolimits_{i=1}^{m} x_{ij} u_i + \sum\nolimits_{k=1}^{p} y_{kj} v_k + \sum\nolimits_{l=1}^{q} z_{lj} w_l \right)$$

$$\text{s.t.} \sum\nolimits_{j=1}^{n} y_{kj} = 1, \text{ for } k = 1, 2, 3, \ldots, p$$

$$\sum\nolimits_{j=1}^{n} z_{lj} = 1, \text{ for } l = 1, 2, 3, \ldots, q$$

where $p_j = 1$, if d_j is a decision node;

otherwise, $0 < p_j < 1$,

and $x_{ij}, y_{kj}, z_{lj} = 0,1$, for $j = 1, 2, 3, \ldots, n$ (1)

Note that in Eqs. (1), the two constraints require problems and solutions to be assigned to one, and only one, decision situation. Also note that Eqs. (1) are problem dependent in that the objective function changes in relation to the three structural constraints specified in Tables 2 to 4 for each given Decision Network problem under consideration.

3 Solution algorithm

For a small- or medium-sized problem, solving Eqs. (1) is straightforward using a commercial package, such as LINDO. When the problem size becomes large enough to involve thousands of decision makers, problems, solutions, and decision situations, it would be cumbersome to construct the model and solve it through LINDO. An algorithm for solving large models involving sequential decisions under uncertainty proposed by Kirkwood (1993) could be applied. However, that would require the modeler to reconstruct the Decision Network problem into a decision tree. With thousands of variables and parameters, as shown in Table 1, this reconstruction would render the solution algorithm inextricable. Alternatively, we develop here a solution algorithm that is specific for solving large-scale Decision Network problems.

Consider the matrices in Tables 2 to 4 and denote them as $\boldsymbol{D}$, $\boldsymbol{A}$, and $\boldsymbol{S}$ respectively, as shown below:

$$D = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}, A = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{p1} & \cdots & b_{pn} \end{pmatrix},$$

$$S = \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{q1} & \cdots & c_{qn} \end{pmatrix}. \qquad (2)$$

Furthermore, suppose that $\boldsymbol{e}$ and $\boldsymbol{e}_{\mathbf{k}}$ are the unit matrix and the k-th unit matrix, respectively, where the k-th unit vector in the k-th unit matrix is equal to one or zero. Note that $\boldsymbol{e}'$ and $\boldsymbol{e}'_k$ are the transpose of $\boldsymbol{e}$ and $\boldsymbol{e}_k$ respectively. For more details on these symbolic representations of the unit matrix and k-th unit matrix, see Eqs. (3) as follows:

$$e=\begin{pmatrix}1\\ \vdots\\ 1\\ \vdots\\ 1\end{pmatrix}, e'=\left[1\cdots1\cdots1\right], e_k=\begin{pmatrix}0\\ \vdots\\ 1\\ \vdots\\ 0\end{pmatrix}, e'_k=\left[0\cdots1\cdots0\right]. \tag{3}$$

The following steps summarize and describe the solution algorithm.

Step 1: Retrieve the row vectors for the access structure. That is, $\boldsymbol{p}_i = \boldsymbol{e}'_i \cdot \boldsymbol{A}$, where $\boldsymbol{p}_i$ denotes the vector of the i-th row, for $i = 1, 2, 3, \ldots, m$.

Step 2: Identify the number of elements for each row where the values of the elements are equal to 1 to obtain Ω_a decomposed matrices. That is, $\boldsymbol{u}_i = \{\boldsymbol{p}_{ij} = 1$, for $j = 1, 2, 3, \ldots, n\}$, for $i = 1, 2, 3, \ldots p$, where $\boldsymbol{u}_i$ is the set of indices of columns for row i where the element $\boldsymbol{p}_{ij}$ is equal to 1. Note that $\langle u_i \rangle$ is the number of elements in set $\boldsymbol{u}_i$ which by definition is greater than or equal to 1 for all i. Let Ω_a be the number of all combinations of non-zero elements across the rows for the access structure, with one, and only one, non-zero element in each row, and we have:

$$\Omega_a = \langle u_1 \rangle \times \langle u_2 \rangle \times \langle u_3 \rangle \times \cdots \times \langle u_m \rangle \tag{4}$$

Each combination stands for a matrix where each row has one, and only one, element whose value is equal to 1.

Step 3: Decompose the solution structure following Steps 1 and 2 to obtain Ω_s decomposed matrices.

Step 4: For each combination of the decomposed matrices of the access and solution structures, compute the overall expected utility and select the combination that yields the highest overall expected utility as the solution. That is, there are a total of $\Omega_a \times \Omega_s$ combinations of decomposed matrices across the access and solution structures. For each combination, we can compute the overall expected utility by summing the expected utility for each decision node. Mathematically, for each decision (or chance) node, the expected utility is equal to:

$$p_l\left(\sum{}_i u_{il} + \sum{}_j v_{jl} + \sum{}_k w_{kl}\right), \tag{5}$$

where u_{il}, v_{jl}, and w_{kl} are the (dis) utilities for the decision maker (s), problem (s), and solution (s) associated with decision node l in the decision, access, and solution structures, respectively, and p_l is the probability that a decision (or chance) node l obtains.

4 Numerical example

Following Han and Lai (2011), we use the same numerical example here to show how the algorithm works. Assume that $\boldsymbol{D}$, $\boldsymbol{A}$, and $\boldsymbol{S}$ are given as follows:

$$D=\begin{pmatrix}0&1&1&0&1\\1&0&0&1&0\end{pmatrix}, A=\begin{pmatrix}0&1&0&0&1\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix},$$
$$S=\begin{pmatrix}0&1&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&1&0&1\end{pmatrix}.$$

Assume further that the probabilities associated with the five decision (chance) nodes are 1.0, 1.0, 1.0, 0.7, and 0.5, respectively, that the utilities associated with the two decision makers are 0.7 and 0.3, respectively, that the disutilities associated with the three problems are –0.6, –0.5, and –0.7, respectively, and that the utilities associated with the four solutions are 0.6, 0.3, 0.7, and 0.5, respectively. We first show how the access structure is decomposed according to Steps 1 to 3 and finally demonstrate how the solution is obtained.

Step 1: Retrieve the row vectors for the access structure.

$$p_1=\begin{pmatrix}0&1&0&0&1\end{pmatrix}, p_2=\begin{pmatrix}0&0&0&1&0\end{pmatrix},$$
$$p_3=\begin{pmatrix}0&0&1&0&0\end{pmatrix}.$$

Step 2: Identify the number of elements for each row where the values of the elements are equal to 1 to obtain Ω_a decomposed matrices.

$$u_1=\{2,5\}, u_2=\{4\}, u_3=\{3\}.$$

$\Omega_a=2\times1\times1=2$, and we have:

$$A_1=\begin{pmatrix}0&1&0&0&0\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix}, A_2=\begin{pmatrix}0&0&0&0&1\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix}.$$

Step 3: Decompose the solution structure following Steps 1 and 2 to obtain Ω_s decomposed matrices.

$\Omega_s=2\times1\times1\times2=4$, and we have:

$$S_1=\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix}, S_2=\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix},$$

$$S_3=\begin{pmatrix}0&0&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix}, S_4=\begin{pmatrix}0&0&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}.$$

Step 4: For each combination of the decomposed matrices of the access and solution structures, compute the overall expected utility and select the combination that yields the highest overall expected utility as the solution. Take the combination of $\boldsymbol{A}_1$ and $\boldsymbol{S}_3$, for example. Let $U(n_i)$ denote the total expected utility for decision node i, for i = 1, 2, 3, 4, and 5. We have:

$$U(n_1)=1.0\times\begin{pmatrix}0&1\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix}+1.0\times\begin{pmatrix}0&0&0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix}+$$
$$1.0\times\begin{pmatrix}0&1&0&0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix}=0.3+0.0+0.3=0.6\ ;$$

$$U(n_2)=1.0\times\begin{pmatrix}1&0\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix}+1.0\cdot\begin{pmatrix}1&0&0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix}+$$
$$1.0\times\begin{pmatrix}0&0&0&0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix}=0.7-0.6+0.0=0.1\ ;$$

$$U(n_3)=1.0\times\begin{pmatrix}1&0\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix}+1.0\times\begin{pmatrix}0&0&1\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix}+$$
$$1.0\times\begin{pmatrix}1&0&0&1\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix}=0.7-0.7+(0.6+0.5)=1.1\ ;$$

$$U(n_4)=0.7\times\begin{pmatrix}0&1\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix}+0.7\times\begin{pmatrix}0&1&0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix}+$$
$$0.7\times\begin{pmatrix}0&0&1&0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix}=0.21-0.35+0.49=0.35\ ;$$

$$U(n_5)=0.5\times\begin{pmatrix}1&0\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix}+0.5\times\begin{pmatrix}0&0&0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix}+$$
$$0.5\times\begin{pmatrix}0&0&0&0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix}=0.35+0.0+0.0=0.35\ .$$

Therefore, for the combination of $\boldsymbol{A}_1$ and $\boldsymbol{S}_3$, the overall expected utility across the five decision nodes is 0.6 + 0.1 + 1.1 + 0.35 + 0.35 = 2.5. The overall expected utilities for other combinations of the access and solution structures can be computed in a similar way. The best "plan," or combination, of the access and solution structures is the one that yields the highest overall expected utility, which in this example is $\boldsymbol{A}_2\boldsymbol{S}_1$ or $\boldsymbol{A}_2\boldsymbol{S}_3$, which yield an overall expected utility of 2.8.

5 Discussion

One might ask what the benefits are of using

Decision Network compared to a decision tree in solving complex problems. The following hypothetical planning problem shows how Decision Network and a decision tree frame the problem differently while coming up with the same answer (Hopkins, 2001). Consider a residential construction project consisting of two decisions: infrastructure and housing constructions. These two decisions fall within the authority of two decision makers, namely the infrastructure provider and the housing builder. Assume that the infrastructure provider could construct either a high-density system of 500 units on 100 acres or a low-density system of 500 units on 250 acres; the housing builder could construct a high-density community of 500 dwelling units on 100 acres or a low-density community of 200 dwelling units on 100 acres.

Based on decision tree analyses, Hopkins (2001) was able to demonstrate algebraically that making plans by simultaneously considering the two decisions, namely infrastructure and housing decisions, yields more benefits in monetary terms than making these decisions independently. Without delving into detailed numerical calculations, we show here that considering multiple stakeholders in a Decision Network framework reinforces Hopkins's argument that making multiple, linked decisions matters. First, the infrastructure decision node can be represented by the diagram shown in Figure 1.

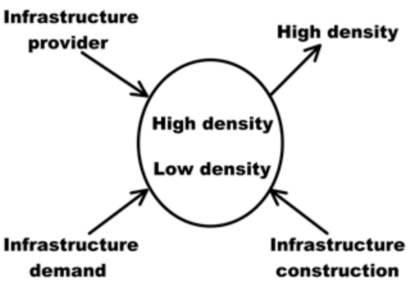

Fig. 1 The Decision Network diagram for the infrastructure decision.

In Figure 1, the circle denotes the decision situation, in which there are two options: high-density or low-density development. The decision maker (infrastructure provider), the problem (infrastructure demand), and the solution (infrastructure construction) are associated with the three inward arrows, whereas the arrow emanating from the decision situation represents the outcome—that is, high-density development in this case, according to Hopkins's original calculations (Hopkins, 2001). Similarly, the housing decision represented in the Decision Network framework is shown in Figure 2. Note that the housing builder should choose low-density development, if the situation is considered independently, given a set of unit costs and revenues for different types of development (Hopkins, 2001).

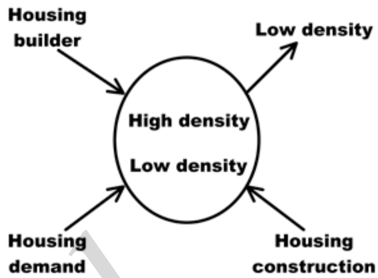

Fig. 2 The DN diagram for the housing decision.

Figure 3 shows that when the two decisions are considered simultaneously, the decision outcome for the infrastructure changes from high-density to low-density development, whereas the decision outcome for the housing builder remains the same.

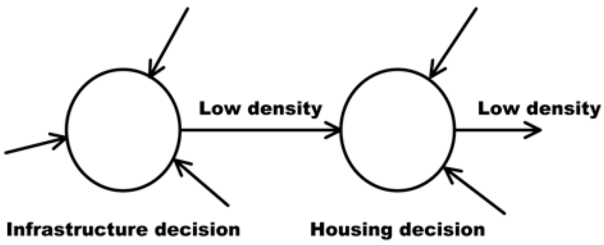

Fig. 3 The DN diagram for the infrastructure and housing decisions combined.

Note that, in Figure 3, the outcome of the infrastructure decision becomes one of the four inputs of the housing decision. Compared with the decision tree representation, the Decision Network diagram is more succinct, more flexible, and richer, while conveying more information. For example, in the two diagrams for the infrastructure and housing decisions, we can add more information about the options inside the decision situations, the infrastructure and housing demands, and the infrastructure and housing supplies without altering the diagrams significantly, which is not the case in a decision tree analysis. These advantages become more significant when the size of the problem is large and involves hundreds or thousands of decision nodes. Of course, a deductive comparison of effectiveness between the Decision Network and decision tree frameworks begs future work.

We have not delved into the multi-attribute preferences that characterize most planning problems, but a straightforward extension of the exiting formulation of Decision Network would, in theory, suffice to cover this issue. One way of doing this is to replace the unidimensional utilities, that is, u_i, v_j, and w_k, with multi-dimensional utilities (Keeney and Raiffa, 1993). Considerable literature exists on multi-attribute decision making, which we cannot delve into here because of limited space. However, the literature provides a basis for future exploration of incorporating multi-attribute preferences into the Decision Network technology.

Most planning problems deal with spatial issues. In its current form, Decision Network can easily be modified to incorporate the spatial dimension into its formulation. Specifically, we can add a fourth element—locations—into the decision situation, in addition to problems, solutions, and decision makers, together with a spatial structure linking those locations to decision situations. This has been done in an attempt to simulate how an urban system works, based on the garbage can model that was originally proposed by Cohen *et al*. (1972) and Lai (2006). The solution algorithm introduced here remains the same, regardless of the addition of the spatial dimension to the Decision Network formulation.

It is arguably true that, because the real world is non-linear, the linear model of Decision Network described in the 0-1 integer program in Eqs. (1) is far from being realistic. We agree that the model does not map reality faithfully, since reality is far more complex than what can be described mathematically. However, the 0-1 integer program simplifies the real problem and serves as a good, approximate basis from which promising solutions can be derived through means other than mathematics, such as graphic and verbal communications. In other words, to take advantage of the problem-solving logic presented in this paper, Decision Network could be developed into a fully-fledged technology to incorporate means of problem solving, whether mathematical or non-mathematical, in dealing with complex planning problems.

6 Conclusions

Planning problems are characterized by difficulty and complexity. Traditional decision analysis techniques that are commonly used by planners are overwhelmingly focused on making independent decisions for single decision makers. Effective planning tools must address multiple stakeholders and multi-attribute preferences at the same time. Here, we provide a theoretical basis for Decision Network, which attempts to make multiple, linked decisions with multiple stakeholders and multi-attribute preferences. In its current form, Decision Network is by no means a mature planning tool because much ambiguity remains to be worked out. With sufficient and persistent effort, the theoretical foundation introduced here could serve as a starting point for development into a fully-fledged technology that helps planners deal confidently with challenging planning problems.

References

Cohen M. D., March J. G., Olsen J. P., 1972. A garbage can model of organizational choice. *Admin Sci Quart*., **17**(1):1–25.

Friend J. K., Hickling A. (Eds), 2005. Planning under pressure: the strategic choice approach (3rd ed). Oxford: Elsevier.

Ghallab M., Nau D., Traverso P., 2004. Automated planning: theory and practice (1st ed). Oxford: Elsevier.

Han H. Y., Lai S. K., 2011. Decision network: a planning tool for making multiple, linked decisions. Environ Plann B., **38**(1):115–28.

Han H. Y., Lai S. K., 2012. Reformulation and assessment of the inventory approach to urban growth boundaries. *Land Use Policy*, **29**(2):351–6.

Hopkins L. D., 1984. Evaluation of methods for exploring ill-defined problems. *Environ Plann B*., **11**(3):339–48.

Hopkins L. D., 2001. Urban development: the logic of making plans. London: Island Press.

Keeney R. L., 2004. Making better decision makers. *Decision Anal*., **1**(4):193–204.

Keeney R. L., Raiffa H., 1993. Decisions with multiple objectives: references and value tradeoffs. New York: Cambridge University Press.

Kirkwood C. W., 1993. An algebraic approach to formulating and solving large models for sequential decisions under uncertainty. *Manage Sci*., **39**(7):900–13

Kirkwood C. W., 1996. Strategic decision making: multi-objective decision analysis with spreadsheets. 1st ed. New York: Duxbury Press.

Lai S. K., 2006. A spatial garbage-can model. *Environ Plann B*., **33**(1):141–56.

Pollock J. L., 2006. Thinking about acting: logical foundations for rational decision making. Oxford: Oxford University Press.